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Dynamic analysis of system with deterministic and stochastic viscoelastic dampers

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Abstract

An analytical method for dynamic analysis of systems with viscoelastic dampers has been developed. Some aspects concerning the critical damping of structure with elastically supported viscoelastic damper are discussed. The considered system is governed by the third order differential equation. The one-sided Green's function for deterministic and stochastic cases is derived in closed analytical form. The free vibration and the forced vibration due to half sine impulse loading have been considered. The analytical solution is derived using Green's function and the Laplace transform method. The numerical results are obtained on the basis of MATHEMATICA system.

1. Introduction

Vibrations of the structures play an increasing role in engineering planning due to the heavier dynamic loads and due to increased environmental consciousness of people. For the analysis of structure vibrations it is necessary to know which elements exist in the area of interest [1]. Viscoelastic dampers can effectively reduce response of the structure to dynamic loads. Using a mathematical model of the structure with dampers one can analyze the damping effects of the system. In the case that the effect of the damper support stiffness is not negligible, one obtains the initial value problem for the third order ordinary differential equation (TODE). The purpose of this paper is to derive and analyse the critical damping coefficients of the TODE. Next, the paper deals with the case, that the damping coefficient is a random function of time variable. The properties of the system are governed by the Green's function. The paper discloses the influence of the randomness of the damping coefficient on the shape of the one-sided Green's function. The

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solution for the deterministic model with time harmonic, Heaviside and other forms of excitation function, that cannot be produced in a real situation, can be found in the existing literature. The response of the considered system due to more realistic excitation, i.e., short duration half sine impulse loading is considered in this paper.

2. Problem formulation

Let the structural system be simplified as a mass m with the stiffness k_f, the stiffness and damping coefficient of the viscoelastic dampers k_d and c_d, respectively, and the stiffness of the member supporting the viscoelastic damper k_b (Fig. 1). The equation of motion for the structure excited by an external force, $f(t)$, can be derived from the following equations [2]:

$$m\ddot{x} + k_f x + P(t) = f(t), \tag{1}$$

$$P(t) = k_b z, \tag{2}$$

$$P(t) = k_d(x - z) + c_d(\dot{x} - \dot{z}), \tag{3}$$

where x and z denote the displacements, $P(t)$ denotes the internal force. The over dot indicates the differentiation with respect to time.

In engineering applications one has to take into account the significant uncertainty with respect to the damping coefficient c_d. In the stochastic approach the random damping coefficient is incorporated into the analysis as a random function of time

$$c_d = c_{d0}(1 + \varepsilon(t, \gamma)), \tag{4}$$

where $\varepsilon(t, \gamma)$ is the zero mean value dimensionless random function ($\langle \varepsilon(t, \gamma) \rangle = 0$) and γ is the elementary event in a complete probability space [3]. The brackets $\langle \cdot \rangle$ in this paper denote an ensemble average and for the sake of conciseness the probability variable γ will be dropped. If the size of randomness is relatively minor, the perturbation theory (small parameter approach) can be an adequate procedure. According to Adomian's remarks [4], in Section 4.2 no assumption concerning the size of the randomness is necessary. If randomness is small, the Adomian's decomposition method leads to perturbation results. The wide class of the random function $\varepsilon(t)$

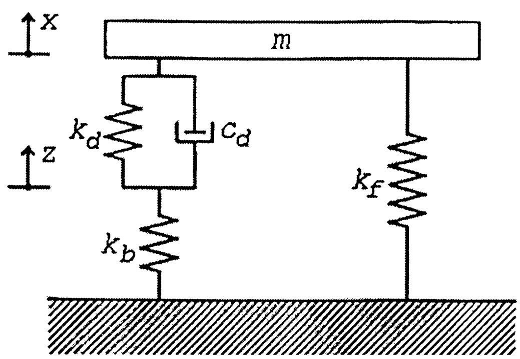

Fig. 1. Model of the analysed dynamic system.

can be described, in the correlation theory, by the correlation function

$$K(t, t_1) = s_d^2 \exp(-\beta|t - t_1|) \sum_{k=0}^{n-1} \frac{\Gamma(n)(2\beta|t - t_1|)^{n-1-k}}{(2n-2)!k!\Gamma(n-k)}, \tag{5}$$

where s_d is the standard deviation and β controls the correlation length. For $n = 1$ one obtains the exponential correlation function, which is commonly used in stochastic dynamics.

Eliminating $P(t)$ and $z(t)$ in Eqs. (1)–(3), leads to the equation of motion of the system as

$$\dddot{x} + \frac{k_b + k_d}{c_{d0}}\ddot{x} + \frac{(k_b + k_f)}{m}\dot{x} + \frac{(k_f k_b + k_b k_d + k_d k_f)}{m c_{d0}} x = F(t), \tag{6}$$

where

$$F(t) = \frac{1}{m}\left[\frac{(k_b + k_d)}{c_{d0}} f(t) + \dot{f}(t)\right] - \left(\dddot{x} + \frac{(k_b + k_f)}{m}\dot{x} - \frac{1}{m}\dot{f}\right)\varepsilon(t). \tag{7}$$

3. Free vibrations: deterministic approach

In the deterministic case ($\varepsilon(t) = 0$) the equation of motion for free vibrations ($f(t) = 0$) is given by

$$\dddot{x} + \frac{A}{c_d}\ddot{x} + \frac{B}{m}\dot{x} + \frac{C}{m c_d} x = 0. \tag{8}$$

The characteristic equation associated with Eq. (8) can be written as

$$\lambda^3 + \frac{A}{c_d}\lambda^2 + \frac{B}{m}\lambda + \frac{C}{m c_d} = 0, \tag{9}$$

where

$$A = k_b + k_d, \quad B = k_b + k_f, \quad C = k_b k_d + k_d k_f + k_f k_b. \tag{10}$$

Eq. (9) leads to discriminant of the form

$$\Delta = q^2 - p^3 \tag{11}$$

where

$$p = \left(\frac{A}{c_d}\right)^2 - \frac{3B}{m}, \quad q = -\left(\frac{A}{c_d}\right)^3 + \frac{9AB}{2mc_d} - \frac{27C}{2mc_d}. \tag{12}$$

The damping coefficients satisfying equation

$$\Delta = 0 \tag{13}$$

are the critical damping coefficients. Substituting Eqs. (12) into Eq. (13) leads to

$$4B^3 c_d^4 - m(A^2B^2 + 18ABC - 27C^2)c_d^2 + 4m^2A^3C = 0. \tag{14}$$

The conditions for existence and nonnegativity of the roots c_d of Eq. (14) are

$$(A^2B^2 + 18ABC - C^2)^2 - 64A^3B^3C \geqslant 0, \tag{15}$$

$$A^2B^2 + 18ABC - 27C^2 \geqslant 0. \tag{16}$$

Conditions (15) and (16) can be written as

$$(AB - 9C)^3(AB - C) \geqslant 0, \tag{17}$$

$$AB(AB - 9C) + 27C(AB - C) \geqslant 0 \tag{18}$$

Hence, the domain where conditions (17) and (18) are fulfilled is

$$AB - 9C \geqslant 0 \tag{19}$$

which becomes

$$k_b^2 - 8(k_bk_d + k_dk_f + k_fk_b) \geqslant 0. \tag{20}$$

If condition (20) is satisfied, the critical damping coefficients $c_{d1} > 0$ and $c_{d2} > 0$ exist and those values can be obtained from Eq. (14).

To find the solution of Eq. (8), as a first step, one has to check if inequality (20) is fulfilled. The next step is to solve Eq. (14) and find the critical damping coefficients c_{d1} and c_{d2}. The general solution of Eq. (8) can be written as

$$x = C_1\varphi_1(t) + C_2\varphi_2(t) + C_3\varphi_3(t), \tag{21}$$

where $\varphi_i(t)$ are the fundamental solutions of Eq. (8) and C_i are arbitrary constants.

Depending on the values of parameters m, k_f, k_b and k_d, there are three cases where c_d can be situated:

Case 1: $0 < c_d < c_{d1}$.
Case 2: $c_{d1} < c_d < c_{d2}$.
Case 3: $c_{d2} < c_d$.

Assume here the following temporary initial conditions:

$$x(0) = Y_1, \quad \dot{x}(0) = Y_2, \quad \ddot{x}(0) = Y_3. \tag{22}$$

For cases 1 and 3 the characteristic equation (9) has one real root and two complex roots:

$$\lambda_1 = l < 0, \quad \lambda_{2,3} = a \mp b\mathrm{i}, \tag{23}$$

where $a < 0,\ b < 0$. The general solution of Eq. (8) in these cases is

$$x = C_1 \mathrm{e}^{lt} + (C_2 \cos bt + C_3 \sin bt)\mathrm{e}^{at} \tag{24}$$

and taking into account the initial conditions (22), one obtains

$$C_1 = \frac{(a^2 + b^2)Y_1 - 2aY_2 + Y_3}{(a - l)^2 + b^2}, \quad C_2 = -\frac{l(2a - l)Y_1 - 2aY_2 + Y_3}{(a - l)^2 + b^2},$$
$$C_3 = \frac{l(a^2 - b^2 - al)Y_1 + (b^2 - a^2 + l^2)Y_2 + (a - l)Y_3}{((a - l)^2 + b^2)b}. \tag{25}$$

For case 2, the characteristic equation (9) leads to three real roots:

$$\lambda_1 = l_1 < 0, \quad \lambda_2 = l_2 < 0, \quad \lambda_3 = l_3 < 0. \tag{26}$$

The general solution of Eq. (7) in this case can be expressed as

$$x = C_1 e^{l_1 t} + C_2 e^{l_2 t} + C_3 e^{l_3 t} \tag{27}$$

and taking into account the initial conditions (22) one obtains

$$C_1 = \frac{l_2 l_3 Y_1 - (l_2 + l_3) Y_2 + Y_3}{(l_3 - l_1)(l_2 - l_1)}, \quad C_2 = -\frac{l_1 l_3 Y_1 - (l_1 + l_3) Y_2 + Y_3}{(l_2 - l_1)(l_3 - l_2)},$$
$$C_3 = \frac{l_1 l_2 Y_1 - (l_1 + l_2) Y_2 + Y_3}{(l_3 - l_2)(l_3 - l_1)}. \tag{28}$$

The initial conditions

$$Y_1 = Y_2 = 0, \quad Y_3 = 1 \tag{29}$$

lead to one-sided Green's function $x(t) = G(t)$, which describes the properties of system (8). For cases 1 and 3 one obtains

$$G(t) = \frac{1}{(a-l)^2 + b^2}\left(e^{lt} - e^{at}\cos bt + \frac{a-l}{b}e^{at}\sin bt\right) \tag{30}$$

and for case 2:

$$G(t) = \frac{1}{(l_3 - l_1)(l_2 - l_1)}e^{l_1 t} - \frac{1}{(l_2 - l_1)(l_3 - l_2)}e^{l_2 t} + \frac{1}{(l_3 - l_1)(l_3 - l_2)}e^{l_3 t}. \tag{31}$$

The formulation here is aimed at constructing the general solution when the initial conditions Y_1, Y_2, Y_3 are viewed as integration constants.

4. Stochastic damping coefficient

In case the damping coefficient is a random function, the stochastic equation of motion of the form

$$\dddot{x} + \frac{A}{c_{d0}}\ddot{x} + \frac{B}{m}\dot{x} + \frac{C}{mc_{d0}}x = -\left(\dddot{x} + \frac{B}{m}\dot{x}\right)\varepsilon(t) \tag{32}$$

is considered. Eq. (32) can be written in the integral-differential form [4]

$$x(t) = C_1\varphi_1(t) + C_2\varphi_2(t) + C_3\varphi_3(t) - \int_0^t G(t-\tau)\left[\dddot{x}(\tau) + \frac{B}{m}\dot{x}(\tau)\right]\varepsilon(\tau)\,d\tau, \tag{33}$$

where $G(t-\tau)$, $\varphi_i(t)$ and C_i, $(i = 1,2,3)$, are given in Eqs. (24), (30) and (27), (31), respectively, depending on cases 1,2 or 3. Taking into account Eq. (33) and the conditions (29), one obtains

$$x = G(t) - \int_0^t G(t-\tau)\left(\dddot{x}(\tau) + \frac{B}{m}\dot{x}(\tau)\right)\varepsilon(\tau)\,d\tau. \tag{34}$$

In further considerations the following expressions will be used:

$$\dot{x} = \dot{G}(t) - \int_0^t \dot{G}(t-\tau)\left(\dddot{x}(\tau) + \frac{B}{m}\dot{x}(\tau)\right)\varepsilon(\tau)\,\mathrm{d}\tau, \tag{35}$$

$$\dddot{x} = \dddot{G}(t) - \left(\dddot{x}(t) + \frac{B}{m}\dot{x}(t)\right)\varepsilon(t) - \int_0^t \dddot{G}(t-\tau)\left(\dddot{x}(\tau) + \frac{B}{m}\dot{x}(\tau)\right)\varepsilon(\tau)\,\mathrm{d}\tau. \tag{36}$$

4.1. First order smoothing approximation

Combining Eqs. (32)–(36), averaging and using the first order smoothing approximation [3]

$$\langle x(t)\varepsilon(t)\varepsilon(\tau)\rangle \cong \langle x(t)\rangle K(t-\tau), \tag{37}$$

yields

$$(1-s^2)\langle \dddot{x} \rangle + \frac{A}{c_{d0}}\langle \ddot{x}\rangle + \frac{B}{m}(1-s_d^2)\langle \dot{x}\rangle + \frac{C}{mc_{d0}}\langle x\rangle$$
$$= \int_0^t \left(\dddot{G}(t-\tau) + \frac{B}{m}\dot{G}(t-\tau)\right)\left(\langle \dddot{x}(\tau)\rangle + \frac{B}{m}\langle \dot{x}(\tau)\rangle\right)K(t-\tau)\,\mathrm{d}\tau \tag{38}$$

where the relations

$$K(t-\tau) = \langle \varepsilon(t)\varepsilon(\tau)\rangle, \quad K(0) = s_d^2. \tag{39}$$

are assumed.

To find the solution of Eq. (38) with the initial conditions (29), the Laplace transform can be applied. Denoting the function in the transform domain by tilde and defining transform as

$$L\{\langle x(t)\rangle\} = \tilde{x}(p) = \int_0^\infty \mathrm{e}^{-pt}\langle x(t)\rangle\,\mathrm{d}t, \tag{40}$$

one obtains the solution of Eq. (38). The solution in the transform domain admits the following form

$$\tilde{x}(p) = \frac{1 - s_d^2 - \tilde{\psi}(p)}{(1 - s_d^2 - \tilde{\psi}(p))p^3 + \dfrac{A}{c_{d0}}p^2 + \dfrac{B}{m}(1 - s_d^2 - \tilde{\psi}(p))p + \dfrac{C}{mc_{d0}}}, \tag{41}$$

where

$$\tilde{\psi}(p) = L\left\{\left(\dddot{G}(t) + \frac{B}{m}\dot{G}(t)\right)K(t)\right\}. \tag{42}$$

Applying the inverse Laplace transform to Eq. (41) and taking into account the initial conditions (29) results in the approximate average solution

$$\langle x(t)\rangle = G_{sB}(t), \tag{43}$$

where $G_{sB}(t) = L^{-1}\{\tilde{x}(p)\}$ can be treated as the approximate Green's function for the stochastic equation (32).

The explicit analytical expression for Eq. (41) can be obtained using the MATHEMATICA system for symbolic computations. To calculate the explicit inverse Laplace transform (43), the values of parameters should be introduced.

4.2. Adomian's decomposition

According to Adomian's decomposition procedure [4], the solution of Eq. (34), with the initial conditions (29), is sought in the form

$$x = x_0(t) + x_1(t) + x_2(t) + \cdots, \tag{44}$$

where $x_0(t) = G(t)$ is the deterministic function, while $x_n(t), (n = 1, 2, \ldots)$, are random functions. Substituting Eq. (44) into Eq. (34) and equating terms of the same order yields

$$x_1(t) = -\int_0^t G(t - t_1)\left(\ddot{x}_0(t_1) + \frac{B}{m}\dot{x}_0(t_1)\right)\varepsilon(t_1)\,\mathrm{d}t_1, \tag{45a}$$

$$x_n(t) = -\int_0^t G(t - t_1)\left(\ddot{x}_{n-1}(t_1) + \frac{B}{m}\dot{x}_{n-1}(t_1)\right)\varepsilon(t_1)\,\mathrm{d}t_1, \quad (n = 2, 3, \ldots). \tag{45b}$$

Each successive $x_n(t)$ depends on the preceding one. Eqs. (45) lead to

$$\dot{x}_1(t) = -\int_0^{t_1} \dot{G}(t_1 - t_2)\left(\dddot{G}(t_2) + \frac{B}{m}\dot{G}(t_2)\right)\varepsilon(t_2)\,\mathrm{d}t_2, \tag{46a}$$

$$\ddot{x}_1(t_1) = \left(\dddot{G}(t_1) + \frac{B}{m}\dot{G}(t_1)\right)\varepsilon(t_1) - \int_0^{t_1} \dddot{G}(t_1 - t_2)\left(\dddot{G}(t_2) + \frac{B}{m}\dot{G}(t_2)\right)\varepsilon(t_2)\mathrm{d}t_2. \tag{46b}$$

Hence, one can obtain an explicit representation of the term $x_n(t)$ in the form of multiple integrals. Substituting Eqs. (46) into (45b) (for $n = 2$), and averaging yields

$$\begin{aligned}\langle x_2(t)\rangle = {} & s_d^2 \int_0^t G(t - t_1)\left(\dddot{G}(t_1) + \frac{B}{m}\dot{G}(t_1)\right)\mathrm{d}t_1 \\ & + \int_0^t \int_0^{t_1} G(t - t_1)\left(\dddot{G}(t_1 - t_2) + \frac{B}{m}\dot{G}(t_1 - t_2)\right)\left(\dddot{G}(t_2) + \frac{B}{m}\dot{G}(t_2)\right)K(t_1 - t_2)\mathrm{d}t_2\,\mathrm{d}t_1.\end{aligned} \tag{47}$$

Combining Eqs. (44), (46a) and (47) leads to the approximate average solution

$$\langle x(t)\rangle = G_{sA}(t), \tag{48}$$

where $G_{sA} = G(t) + \langle x_2(t)\rangle$. Expression (48) can be treated as the approximate Green's function for the stochastic equation (32).

A similar procedure as for the average solution can be used to obtain the second order approximation of the variance function $V_x(t) = \langle x_1^2(t)\rangle + \langle x_2^2(t)\rangle - \langle x_2(t)\rangle^2$. To ascertain validity of the second order approximate solution, one can resort to Monte Carlo simulation.

5. Forced vibrations

The solution of the deterministic nonhomogeneous equation of motion ($\varepsilon(t) = 0$, hence $c_{d0} = c_d$)

$$\dddot{x} + \frac{A}{c_d}\ddot{x} + \frac{B}{m}\dot{x} + \frac{C}{mc_d}x = f_l(t), \tag{49}$$

where

$$f_l(t) = \frac{1}{m}\left[\frac{A}{c_d}f(t) + \dot{f}(t)\right] \tag{50}$$

with the initial conditions (22) can be represented as

$$x = C_1\varphi_1(t) + C_2\varphi_2(t) + C_3\varphi_3(t) + \int_0^t G(t-\tau)f_l(\tau)\,\mathrm{d}\tau, \tag{51}$$

where $\varphi_i(t)$ and C_i are given in Eqs. (24), (25) and (27), (28), respectively, depending on cases 1, 2 or 3.

A variety of viscous dampers, forming parts of a structural system, have been developed to reduce building responses to dynamic loads such as earthquakes.

Consider the dynamic response of the system exposed to impulse loading of the form

$$f(t) = \begin{cases} p_0 \sin\dfrac{\pi}{t_0}t, & 0 \leqslant t \leqslant t_0, \\ 0, & t > t_0. \end{cases} \tag{52}$$

Substituting Eq. (52) in Eq. (50) leads to

$$f_l(t) = \frac{p_0}{m}(1 - \mathrm{H}(t-t_0))\left(\frac{A}{c_d}\sin\frac{\pi t}{t_0} + \frac{\pi}{t_0}\cos\frac{\pi t}{t_0}\right), \tag{53}$$

where $\mathrm{H}(t-t_0)$ is the Heaviside function.

Once specific values of $A, B, C, m, c_d, p_0, t_0$ and the initial conditions Y_1, Y_2, Y_3 are given, the solution for the displacement (Eq. (51)), can be found as

$$x = x_{in}(t) + x_{fo}(t). \tag{54}$$

The first term on the right hand side of Eq. (54), $x_{in}(t)$, represents the response of the system to the initial conditions. The remaining term, $x_{fo}(t)$, is the response of the system due to the forcing function. If the initial conditions Y_1, Y_2, Y_3 are viewed as arbitrary constants, the term $x_{in}(t)$ comprise the general solution of the homogeneous equation corresponding to Eq. (49). For the initial conditions $x(0) = 0$ and $\ddot{x}(0) = 0$, Eqs. (49) and (53) lead to:

for cases 1 and 3 one obtains

$$x_{in}(t) = \frac{1}{(a-l)^2 + b^2}\left[-2aY_2(\mathrm{e}^{lt} - \mathrm{e}^{at}\cos bt) + \frac{1}{b}(b^2 - a^2 + l^2)Y_2\mathrm{e}^{at}\sin bt\right], \tag{55}$$

$$x_{fo}(t) = \frac{p_0}{m}\cdot\int_0^t G(t-\tau)(1 - \mathrm{H}(\tau - t_0))\left(\frac{A}{c_d}\sin\frac{\pi\tau}{t_0} + \frac{\pi}{t_0}\cos\frac{\pi\tau}{t_0}\right)\mathrm{d}\tau, \tag{56}$$

where $G(t-\tau)$ is given in Eq. (30);

for case 2

$$x_{in}(t) = -\frac{(l_2 + l_3)Y_2}{(l_2 - l_1)(l_3 - l_1)}e^{l_1 t} + \frac{(l_1 + l_3)Y_2}{(l_3 - l_2)(l_2 - l_1)}e^{l_2 t} - \frac{(l_1 + l_2)Y_2}{(l_3 - l_2)(l_3 - l_1)}e^{l_3 t} \tag{57}$$

and the second term, $x_{fo}(t)$, assumes the form of Eq. (56) with $G(t-\tau)$ given in Eq. (31).

The initial value problem described in Eqs. (49) and (53) can also be solved by the Laplace transform method. The solution in the transform domain can be written as

$$\tilde{x}(p) = \frac{(p + A/c_d)Y_2}{D_3(p)} + \frac{(\pi p_0/mt_0)(p + A/c_d)(1 + e^{-t_0 p})}{(p^2 + (\pi/t_0)^2)\cdot D_3(p)}, \tag{58}$$

where

$$D_3(t) = p^3 + Ap^2/c_d + Bp/m + C/mc_d. \tag{59}$$

For specified parameters, the inverse Laplace transform, $L^{-1}\{\tilde{x}(p)\}$, can be calculated by the MATHEMATICA system.

6. Numerical results and conclusions

In this paper the response of the system with elastically supported viscoelastic damper for free vibration and for forced vibration in the form of half sine impulse is derived analytically. An exact solution to this problem has been obtained by employing the Laplace transform method and Green's function method, respectively. The numerical calculations were carried out for structure exposed to short duration impulse loading.

For numerical examples the following values of parameters have been specified: $m = 10^6$ kg, $k_b = 2.0 \times 10^9$ N/m, $k_d = 3.0 \times 10^7$ N/m, $k_f = 2.0 \times 10^8$ N/m. Condition (20) is satisfied. Two positive critical damping coefficients bounding underdamped vibration and overdamped vibration exist and are equal to $c_{d1} = 2.45305 \times 10^7$ Ns/m and $c_{d2} = 2.46659 \times 10^7$ Ns/m.

To clarify the vibration characteristics of the system with various damping coefficients, time histories of free vibration and forced vibration, respectively, have been computed for three cases. Fig. 2 shows the displacement $x(t)$ for $c_d = 10^7$ Ns/m $\in (0, c_{d1})$, $c_d = 2.46 \times 10^7$ Ns/m $\in (c_{d1}, c_{d2})$ and $c_d = 10^8$ Ns/m $\in (c_{d2}, \infty)$, respectively.

Case 1 corresponds to underdamped vibration. The displacement in case 2 is overdamped and it does not represent the vibrating motion. The range of overdamped vibration is finite, in contrast to semi-infinite for second order differential equation. In case 3, a vibration component is superposed on an overdamped case.

Figs. 3(a)–(c) show the effect of the standard deviation s_d of the damping coefficient on the Green's function $G_{sB}(t)$ for cases 1, 2 and 3, respectively. In the real systems, the stochastic inhomogeneity is often present and is likely to increase the damping characteristics in a significant way. For comparison, the results are presented for the exponential correlation function for $\beta = 1/s$ and $s_d = 0$ (solid line, deterministic case), $s_d = 0.6$ (dashed line) and $s_d = 0.9$ (dotted line). Increasing the variance of damping coefficient leads to decreasing the maximum displacement amplitude, and in case 2 it can lead to vibrating motion (see Fig. 3(b)).

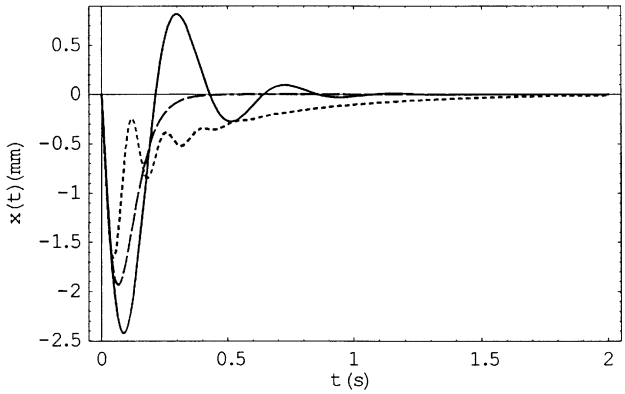

Fig. 2. Free vibration for three cases: $c_d = 10^7$ Ns/m (solid line), $c_d = 2.46 \times 10^7$ Ns/m (dashed line), $c_d = 10^8$ Ns/m (dotted line).

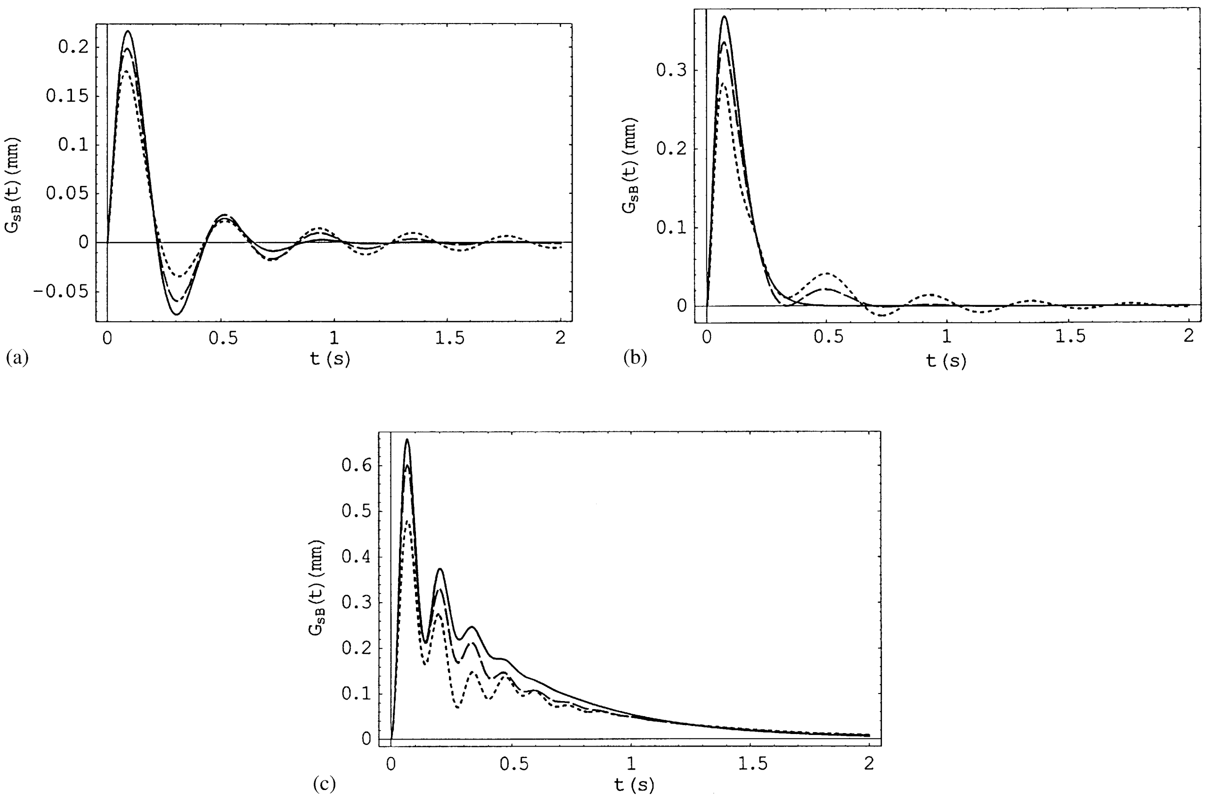

Fig. 3. Green's function: (a) case 1, (b) case 2, (c) case 3, for standard deviation: $s_d = 0$ (solid line), $s_d = 0.6$ (dashed line), $s_d = 0.9$ (dotted line).

The forced displacement field, $x(t)$, due to half sine impulse loading of the amplitude $p_0 = 10^5$ N, related to cases 1, 2 and 3, is shown in Figs. 4(a)–(c), respectively, for duration $t_0 = 0.1$ s (dotted line), $t_0 = 0.5$ s (dashed line) and $t_0 = 1$ s (solid line). Here, in case 3, the value of

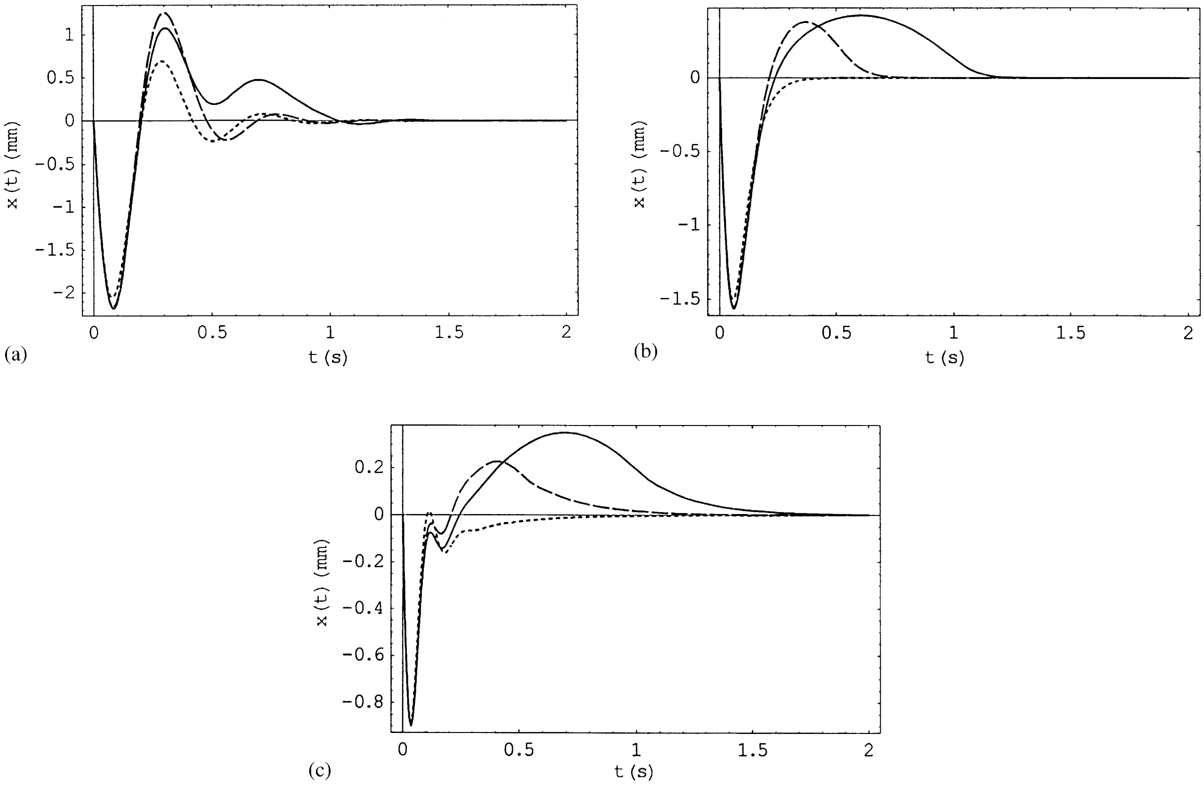

Fig. 4. Displacement due to impulse loading: (a) case 1, (b) case 2, (c) case 3, for duration: $t_0 = 0.1$ s (dotted line), $t_0 = 0.5$ s (dashed line), $t_0 = 1$ s (solid line).

$c_d = 5.0 \times 10^7$ Ns/m is taken. To analyse the displacement field, $x(t)$, the initial displacement has been specified to zero and the initial velocity $Y_2 = -0.05$ ms^{-1}. It should be noted that the smaller critical damping coefficient, c_{d1}, is practically meaningful.

The proposed method is useful for dynamic analysis of structures and derived expressions can serve as a benchmark solution. Various approximate numerical results, obtained on the basis of finite element method, can be confronted with the derived analytical solution.

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